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EFFECT OF COULOMB COLLISIONS, LANDAU DAMPING AND PARTICLE TRAPPING ON LINEAR AND NONLINEAR ION ACOUSTIC WAVES

Chung Chan

Northeastern University
Department of Electrical Engineering
Boston, MA 02115

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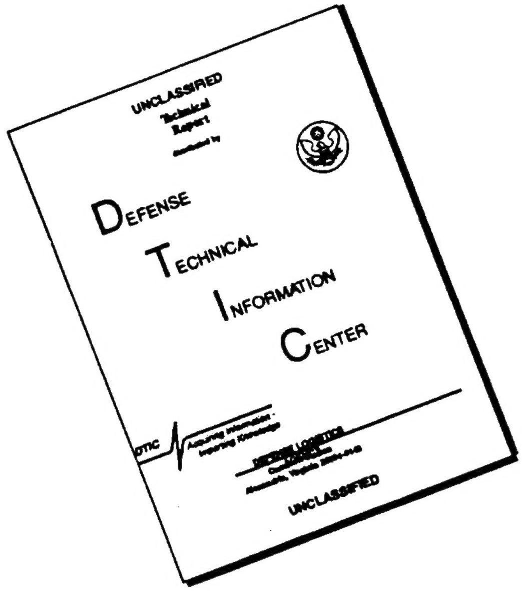

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The general propagator expansion method is used for the study of the collision effects and the Fokker-Planck kinetic equation is employed for several collisional processes. In drift-free plasma, the studies show that the plasma may become unstable if *e-i* collision growth can overcome Landau damping and other collision damping. The contamination can affect ion acoustic waves significantly and a small amount of light ion contamination can cause much higher damping. For our experiment on the ion acoustic waves in drift-free plasma, the experimental results are explained with the consideration of collision and contamination effects and are close to our theoretical results.For a plasma with a quasi-static electric field, two ion acoustic modes may propagate in a single ion species plasma. This ion-acoustic instability associated with the slow mode is a kinetic instability and is caused by the interaction of ions and electrons. In our experiment, the fast mode is found and the measured results agree with the theoretic values. In a two-ion-species plasma, the velocity differences between the ion species can cause the ion-ion two-stream instability. This is a fluid instability and only exists in a range of ion drift velocities and wave frequencies. An experiment was performed to study ion-ion two-stream instability. The unstable waves generated by the ion beams are observed as predicted by theory and coincide with the kinetic dispersion relations for the ion-ion two stream instability.

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1. Introduction

Ion acoustic waves, which were first predicted by *Tonks and Langmuir* [1929] using the fluid analysis, have been studied by a number of authors on the basis of the Vlasov equation. In 1946, Landau gave a method for solving the linearized Vlasov equation which indicated that the ion acoustic waves are damped [*Landau*, 1946]. This damping effect is caused by the negative slope of the ion distribution function for resonant velocities comparable to the phase velocity of the waves. Generally, ion acoustic waves are restricted to the long wavelength domain because of strong ion Landau damping at short wavelength. Ion Landau damping effects are, therefore, most important wherever the phase velocity of the ion acoustic wave is comparable to the ion thermal velocity. Experimentally, the ion acoustic waves were first observed by *Revans* [1933], and their Landau damping was measured by *Wong et al.* [1964].

The plasmas in the low attitude of the ionosphere and also in the experiments are usually collisional. There have been many investigations of a collisional plasma in the presence or absence of an external electric field. The collisional damping of the ion waves was studied by *Bhadra and Varma* [1964]; they used BGK equation to describe the ion-ion collisions. *Stéfant* [1971] investigated the influence of electron-ion collisions on the ion acoustic waves in the present of a longitudinal current with a BGK collision integral and found that the electron-ion collisions facilitate the ion acoustic oscillations. *Kulsrud and Shen* [1966] and *Ono and Kulsrud* [1975] used a more accurate method for the collisional process and found that ion-ion collisions damped the ion-acoustic waves. In their method, the ions were described by the Fokker-Planck equation and the electrons were treated by a fluid equation. *Buti* [1968] presented a more realistic model and used complicated Fourier-Laplace transforms to study the ion acoustic waves in a collisional plasma. In the model, both the electron and the ions are treated by Fokker-Planck equation and it is found that the electron-ion collisional undamping was small compared to the ion-ion collisional damping for a wide range of wave numbers and particle temperature. *Jasperse* [1984] gave a general propagator expansion method for solving linearized plasma kinetic problems with collisions and *Basu and Jasperse* [1988] used this method to solve the collisional plasma based on linearized Balescu-Lenard kinetic equations, which shows that the electron-ion collisional undamping effect may overcome the ion-ion, electron-electron, and ion-electron collisional effects. So unlike a collisionless drift-free plasma, where the ion acoustic waves are always Landau damped and the plasma is stable, the collisional plasma may be unstable when the collisional undamping rate exceeds the electron and ion Landau damping rates.

Plasma in planetary ionospheres consists of may kinds of ions, and the ion composition ratio changes with height. In the experiments, we have also the impurities in the plasma. To study the phonomena which are related to waves in the ionosphere as well as in the experiments, it is important to understand the nature of waves in the multi-ion species plasma. A multi-ion component plasma exhibits some phenomena which do not occur in a

single ion plasma; one of these is the resonance of light ions with the heavier ion mode waves. *Alexeff et al.* [1967], *Nakamura et al.* [1975] and *Nakamura et al.* [1976] experimentally investigated the changes in wave properties of ion acoustic in multi-ion plasma with a variable composition ratio. *Fried et al.* [1971] analytically studied the properties of the ion acoustic waves in a multi-ion plasma by expanding the plasma dispersion function. Their studies have indicated that a small amount of light ion contamination will cause higher damping rate than that in single ion species plasma.

In this report, I will study the effects of collisions and contamination on ion acoustic waves. The dispersion relation for collisional plasma will be given in Section 2. The general propagator expansion method presented by *Jasperse* [1984] will be used to solve kinetic dispersion equation with collisions. The collisional processes we consider here will include the collisions between charge neutral particles and between charged particles. The Fokker-Planck equation will be used. The contamination effect will be studied in Section 3 with the consideration of a variety of ion composition ratio and ion- electron temperature ratio. In Section 4, all the effects considered in this report will be put together to analyze the data we measured in the experiments.

2. Dispersion Relation for Collisional Plasma

The kinetic equation in the electrostatic approximation for a collisional plasma is

$$\frac{\partial f_j}{\partial t} + \mathrm{v} \cdot \nabla f_j + \frac{q_j}{m_j} E \cdot \frac{\partial f_j}{\partial \mathrm{v}} = \mathcal{L}(f_j), \tag{1}$$

where **E** is electrostatic field, q_j and m_j are charge and mass of the particle, and $\mathcal{L}(\ )$ is the (non-linear) collision operator that acts on the velocity-space coordinates of the particle distribution function f_j. We seek a perturbation solution of the form

$$f_j(\mathrm{r},\mathrm{v},t) = f_{j0}(\mathrm{v}) + f_1(\mathrm{r},\mathrm{v},t), \tag{2}$$

$$\mathrm{E}(\mathrm{r},t) = \mathrm{E}_1(\mathrm{r},t), \tag{3}$$

where f_0 is the zero-order distribution function and f_1 and $\mathbf{E}_1$ are perturbed quantities. Using Eqs. (2) and (3) to Linearize Eq. (1), we get the first order approximation of the equation

$$\frac{\partial f_{j1}}{\partial t} + \mathrm{v} \cdot \nabla f_{j1} + \frac{q_j}{m_j} \mathrm{E}_1 \cdot \frac{\partial f_{j0}}{\partial \mathrm{v}} = \sum_\sigma L_{j\sigma} f_{\sigma 1}, \tag{4}$$

where $L_{j\sigma}$ is the linearized collision operator on the σth particle and the summation is on all the particle species in the plasma. Assume that the waves are plane waves and all perturbation terms have the factor $e^{i(\mathrm{k}\cdot\mathrm{r}-\omega t)}$. Then Eq. (4) becomes

$$-i\omega f_{j1} + i(\mathrm{k}\cdot\mathrm{v}) f_{j1} - \sum_\sigma L_{j\sigma} f_{\sigma 1} = \frac{i q_j \phi_1}{m_j} \mathrm{k} \cdot \frac{\partial f_{j0}}{\partial \mathrm{v}}. \tag{5}$$

Define the collisional propagator operator as

$$U = \left\{ (-i\omega + i\mathrm{k}\cdot\mathrm{v})\mathrm{I} - \left[L_{j\sigma}\right] \right\}^{-1}, \tag{6}$$

where I is the unit matrix and $[L_{j\sigma}]$ represents a matrix with $L_{j\sigma}$ as its element in row j and column σ. Then we can solve Eq. (5) for all the particle species as

$$\left[f_{j1}\right] = U \cdot \left[\frac{iq_j\phi_1}{m_j}\mathrm{k} \cdot \frac{\partial f_{j0}}{\partial v}\right], \tag{7}$$

where the symbol $[x_j]$ stands for an array with x_j as its jth element.
Using the Poisson's equation

$$k^2\varepsilon_0\phi_1 = \sum_j q_j n_{j0}\int f_{j1}dv. \tag{8}$$

we then have

$$1 = -\frac{i}{k^2\varepsilon_0}\int dv \cdot \left\{\left[q_j n_{j0}\right]' \cdot U \cdot \left[\frac{q_j\phi_1}{m_j}\mathrm{k} \cdot \frac{\partial f_{j0}}{\partial v}\right]\right\}. \tag{9}$$

Since $L_{j\sigma}$ is usually a complicated integro-differential operator, it is difficult to get a closed-form solution for Eq. (9). Here we will use the general propagator expansion method [*Jasperse*, 1984]. The method can be applied to a wide class of collision operators and usually can produce closed-form results for Eq. (9). Here we assume the limit of weak collision i.e., for any j and σ for the linearized collision operator, we have

$$\left|L_{j\sigma}\right| << \left|\omega - \mathrm{k}\cdot\mathrm{v}\right|. \tag{10}$$

Then the collision propagator operator can be expanded into the Taylor series

$$U = \sum_{n=0}^{\infty}\left\{(i\omega - i\mathrm{k}\cdot\mathrm{v})^{-1}\left[L_{j\sigma}\right]\right\}^n \cdot (-\omega + i\mathrm{k}\cdot\mathrm{v})^{-1}. \tag{11}$$

Substituting Eq. (11) into Eq. (9), we get

$$1 = -\frac{1}{k^2\varepsilon_0}\int dv \cdot \left[q_j n_{j0}\right]' \cdot \sum_{n=0}^{\infty}\left\{(\ i\omega - i\mathrm{k}\cdot\mathrm{v}\ )^{-1}\left[L_{j\sigma}\right]\right\}^n\left[\frac{q_\sigma}{m_\sigma}\mathrm{k}\cdot\frac{\partial f_{\sigma 0}/\partial v}{\omega - \mathrm{k}\cdot\mathrm{v}}\right]. \tag{12}$$

Neglect the high-order terms in Eq. (12) and consider the Maxwellian distribution for f_{j0}, we get

$$1 + \sum_j \frac{1}{k^2\lambda^2{}_j}\left\{W_0\left(\frac{\omega - kv_j}{kv_{tj}}\right) + iv_{tj}^2\int dv \frac{1}{\omega - \mathrm{k}\cdot\mathrm{v}}\sum_\sigma \frac{q_\sigma}{q_i}\frac{m_j}{m_\sigma}L_{j\sigma}\frac{1}{\omega - \mathrm{k}\cdot\mathrm{v}}\mathrm{k}\cdot\frac{\partial f_{\sigma 0}}{\partial v}\right\} = 0. \tag{13}$$

In this report, we will use Fokker-Planck collision operator for $\mathcal{L}(\ )$. For the collisions between charges and neutral particles, we use the Brownian form of Fokker-Plank equation [*Fokker*, 1914; *Plank*, 1917; *Chandrasekhhar*, 1943]. The collision operator for the Brownian form is

$$\mathcal{L}(f_j) = \sum_\sigma \nu_{j\sigma}\frac{\partial}{\partial v}\cdot\left(\mathrm{v} + v_{tj}^2\frac{\partial}{\partial v}\right)f_j, \tag{14}$$

and linearized collision operator for the j-n collision is

$$L_{ij} = \nu_{jn}\frac{\partial}{\partial v}\cdot\left(\mathrm{v} + v_{tj}^2\frac{\partial}{\partial v}\right). \tag{15}$$

For the collisions between charged particles, we use the Rosenbluth form of the Fokker-Planck equation [*Rosenbluth et al.*, 1957] which gives the following collision operator

$$\mathcal{L}(f_j) = -\Gamma_j\frac{\partial}{\partial v}\cdot\left(f_j\frac{\partial H_j}{\partial v}\right) + \frac{1}{2}\Gamma_j\frac{\partial^2}{\partial v\partial v}\cdot\left(f_j\frac{\partial^2 G_j}{\partial v\partial v}\right), \tag{16}$$

$$H_j(\mathrm{v}) = \sum_\sigma n_{\sigma 0} \frac{m_j + m_\sigma}{m_\sigma} \left(\frac{q_\sigma}{q_j}\right)^2 \int du f_\sigma(\mathrm{u}) |\mathrm{u} - \mathrm{v}|^{-1},$$

$$G_j(\mathrm{v}) = \sum_\sigma n_{\sigma 0} \left(\frac{q_\sigma}{q_j}\right)^2 \int du f_\sigma(\mathrm{u}) |\mathrm{u} - \mathrm{v}|,$$

$$\Gamma_j = \frac{\omega^4{}_{pj}}{4\pi n^2{}_{j0}} \ln \Lambda_j \qquad \Lambda_j = \frac{12\pi}{g_j} \qquad g_j = \frac{1}{n_{j0} \lambda^3{}_j}$$

and the linearized collision operator for the j-σ collision is composed of four terms

$$L_{j_\sigma} = L^{(1)}{}_{jj} + L^{(2)}{}_{jj} + L^{(1)}{}_{j\sigma} + L^{(2)}{}_{j\sigma}, \tag{17}$$

$$L^{(1)}{}_{jj} = -\Gamma_j n_{\sigma 0} \frac{m_j + m_\sigma}{m_\sigma} \left(\frac{q_\sigma}{q_j}\right)^2 \left(\frac{\partial}{\partial \mathrm{v}} \cdot\right) \left(\frac{\partial}{\partial \mathrm{v}} \int du f_{\sigma 0}(\mathrm{u}) |\mathrm{u} - \mathrm{v}|^{-1}\right)$$

$$L^{(2)}{}_{jj} = \frac{1}{2} \Gamma_j n_{\sigma 0} \left(\frac{q_\sigma}{q_j}\right)^2 \left(\frac{\partial^2}{\partial \mathrm{v} \partial \mathrm{v}} :\right) \left(\frac{\partial^2}{\partial \mathrm{v} \partial \mathrm{v}} \int du f_{\sigma 0}(\mathrm{u}) |\mathrm{u} - \mathrm{v}|\right)$$

$$L^{(1)}{}_{j\sigma} = -\Gamma_j n_{\sigma 0} \frac{m_j + m_\sigma}{m_\sigma} \left(\frac{q_\sigma}{q_j}\right)^2 \left(\frac{\partial}{\partial \mathrm{v}} \cdot\right) f_{j0} \frac{\partial}{\partial \mathrm{v}} \int d\mathrm{u} |\mathrm{u} - \mathrm{v}|^{-1} \int dv \delta(\mathrm{v} - \mathrm{u}),$$

$$L^{(2)}{}_{j\sigma} = \frac{1}{2} \Gamma_j n_{\sigma 0} \left(\frac{q_\sigma}{q_j}\right)^2 \left(\frac{\partial^2}{\partial \mathrm{v} \partial \mathrm{v}} \cdot\right) f_{j0} \frac{\partial^2}{\partial \mathrm{v} \partial \mathrm{v}} \int d\mathrm{u} |\mathrm{u} - \mathrm{v}| \int dv \delta(\mathrm{v} - \mathrm{u})$$

Grewal [1964] gave the relationship between the Brownian form and the Rosenbluth form of the Fokker-Planck equation. Substitute Eqs. (15) and (17) into Eq. (13) and simplify the equation obtained. Since the algebra involved in the simplification is very lengthy, we put it in Appendix. The final result for the dispersion relation to the first order in collisionality for a collisional, two-constituent plasma is

$$1 + \sum_j \frac{1}{k^2 \lambda^2{}_j} \left\{ W_0 \frac{\omega}{k v_{tj}} + \frac{i}{k v_{tj}} \left[-\frac{v_{jn}}{6} \frac{\omega}{k v_{tj}} W_2\left(\frac{\omega}{k v_{tj}}\right) + \sum_\sigma \eta_{j\sigma} S_{j\sigma}\left(\frac{\omega}{k v_{tj}}\right) \right] \right\} = 0 \tag{18}$$

Notice that the terms in the square bracket is caused by the collisions, in which the first term is from the collisions between charged and neutral particles and the second term from the collisions between charges particles. We also studied the simple BGK collision from the BGK equation is similar to what we get here and that the approximation of the general propagator expansion method is very good when the frequency ω is much larger than collision frequency but becomes less accurate when the frequency ω is less than several times of the collision frequency [*Jin*, 1995]. *Basu and Jasperse* [1988] solved Eq. (13) with the Balescu-Lenard collision operator for the collisions between charges particles. By a comparison of our results with theirs, we find that the both results are exactly the same. This is because *Basu and Jasperse* [1988] used approximation $\varepsilon(\omega, \mathbf{k}) \approx 1$ in their calculations. *Liboff* [1990] shows that the Balescu-Lenard equation will reduce to the

Landau form of the Fokker-Planck equation when $e(\omega,\mathbf{k}) \to 1$ and the Rosenbluth form and the Landau form of the Fokker-Planck equation are equivalent to each other. Expanding the left side of Eq. (18), we get the analytic expressions for the phase velocity and the damping coefficient for the ion waves in a collisional plasma:

$$\frac{\omega_r^2}{k^2} = c_s^2 F_{1\omega} \tag{19a}$$

$$\frac{\omega_i}{\omega_{pi}} = \left(\frac{\omega_i}{\omega_{pi}}\right)_L + \left(\frac{\omega_i}{\omega_{pi}}\right)_{en} + \left(\frac{\omega_i}{\omega_{pi}}\right)_{in} + \left(\frac{\omega_i}{\omega_{pi}}\right)_{ei} + \left(\frac{\omega_i}{\omega_{pi}}\right)_{ee} + \left(\frac{\omega_i}{\omega_{pi}}\right)_{ii} + \left(\frac{\omega_i}{\omega_{pi}}\right)_{ie} \tag{19b}$$

$$\left(\frac{\omega_i}{\omega_{pi}}\right)_L = -k\lambda_e\sqrt{\frac{\pi}{8}}F^2{}_{1\omega}(\vartheta)\left[1+\frac{6}{\vartheta}F^{-1}{}_{1\omega}(\vartheta)\right]^{-1} G_{1\omega}(\vartheta),$$

$$\left(\frac{\omega_i}{\omega_{pi}}\right)_{in} = -\frac{1}{2}\frac{\nu_{in}}{\omega_{pi}}\left[1+\frac{6}{\vartheta}F_{1\omega}^{-1}(\vartheta)\right]^{-1},$$

$$\left(\frac{\omega_i}{\omega_{pi}}\right)_{ei} = \frac{1}{4}\frac{\nu_{ei}}{\omega_{pi}}F_{1\omega}(\vartheta)\left[1+\frac{6}{\vartheta}F_{1\omega}^{-1}(\vartheta)\right]^{-1},$$

for the complex ω case, where

$$F_{1\omega}(\vartheta) = \frac{1}{1+k^2\lambda^2{}_e} + \frac{3}{\vartheta}, G_{1\omega}(\vartheta) = \left(\frac{m_e}{m_i}\right)^{1/2} + \vartheta^{\frac{3}{2}} e^{-\frac{\vartheta}{2}F(\vartheta)_\omega}, \text{and} \vartheta = T_e / T_i$$

and

$$\frac{\omega^2}{k_r^2} = c_s^2 F_{1k}(\vartheta) \tag{20a}$$

$$k_i\lambda_\varepsilon = (k_i\lambda_e)_L + (k_i\lambda_e)_{en} + (k_i\lambda_e)_{in} + (k_i\lambda_e)_{ei} + (k_i\lambda_e)_{ee} + (k_i\lambda_e)_{ii} + (k_i\lambda_e)_{ie} \tag{20b}$$

$$(k_i\lambda_e)_L = \frac{\omega}{\omega_{pi}}\sqrt{\frac{\pi}{8}}\left[1+\frac{3}{\vartheta}F^{-2}{}_{1k}(\vartheta)\right]^{-1} G_{1k}(\vartheta),$$

$$(k_i\lambda_e)_{in} = \frac{1}{2}\frac{\nu_{in}}{\omega_{pi}} F_{1k}{}^{-\frac{3}{2}}(\vartheta)\left[1+\frac{3}{\vartheta}F^{-2}{}_{1k}(\vartheta)\right]^{-1},,$$

$$(k_i\lambda_e)_{ei} = -\frac{1}{4}\frac{\nu_{ei}}{\omega_{pi}} F_{1k}{}^{-\frac{1}{2}}(\vartheta)\left[1+\frac{3}{\vartheta}F^{-2}{}_{1k}(\vartheta)\right]^{-1},$$

for the complex k case, where

$$F_{1k}(\vartheta) = 1 - \frac{\omega}{\omega_{\pi}^2} + \frac{3}{\vartheta}, \text{ and } G_{1k}(\vartheta) = \left(\frac{m_e}{m_i}\right)^{1/2} + \vartheta^{\frac{3}{2}} e^{-\frac{\vartheta}{2}F_{1k}(\vartheta)}.$$

The subscripts L in the equations stands for the Landau damping term, and *en, in, ei, ee, ii,* and *ie* stand for the *e-n, i-n, e-i, e-e, i-i,* and *i-e* collision terms, respectively. Usually, the *en* term is much less than the *in* term, and *ee, ii,* and *ie* terms are much less than the *ei* term. Note that, approximately, Landau damping coefficient is proportional to k or ω and the damping coefficients from the collisions are independent of k or ω for the lower k or ω values. Figure 1*a* shows the dispersion relation of ion acoustic waves with *i-n* collisions only while Figure 1*b* shows the dispersion relation of ion acoustic waves with *e-i* collisions

only. The *e-i* collisions contribute an undamping effect on the ion acoustic waves. If this effect is large enough to overcome the Landau damping and other collision damping, the collision-driven ion acoustic instability may occur. Since the Landau damping increases as wave frequency increases, this kind of instability may occur only in the low frequency,. From Eqs. (19b) and (20b) we can get the marginal stability condition for this instability:

$$\frac{\omega}{\omega_{pi}} = \frac{1}{\sqrt{2\pi}}\left[\frac{\nu_{ei}}{\omega_{pi}} F^{-\frac{1}{2}}{}_{1\omega(k)}(\vartheta) - \frac{2\nu_{in}}{\omega_{pi}} F^{-\frac{3}{2}}{}_{1\omega(k)}(\theta)\right] G^{-1}_{1\omega(k)}(\theta) \tag{21}$$

3. Contamination effect

To study the basic characteristics of contamination effect on ion acoustic waves, we will not consider the collision effects here. We will combine the two effects in the next section when we analyze our experimental data. Expanding the plasma dispersion function in Eq. (13) without collisions ($\nu_{jn} = \nu_{js} = 0$) to the first order, we can obtain

$$\frac{\omega_r^2}{k^2} = \frac{c_s^2}{1 + k^2\lambda_e^2} \tag{22a}$$

$$\left(\frac{\omega_i}{\omega_{pi}}\right)_L = \sqrt{\frac{\pi}{8}} k\lambda_e \left(1 + k^2\lambda_e^2\right)^{-2} \left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \sum_j \frac{n_{j0}}{n_{e0}} \left(\frac{m_j}{m_i}\right)^{\frac{1}{2}} \left(\frac{T_e}{T_j}\right)^{\frac{3}{2}} e^{-\frac{1}{2}\frac{T_e}{T_j}\frac{m_j}{m_i}\left(1+k^2\lambda_e\right)^{-1}}\right]$$

(22b)

for the complex ω case, and

$$\frac{\omega^2}{k^2{}_r} = c_s^2\left(1 - \frac{\omega^2}{\omega_{pi}^2}\right) \tag{23a}$$

$$\left(k_i\lambda_e\right)_L = \sqrt{\frac{\pi}{8}} \frac{\omega}{\omega_{pi}} \left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \sum_j \frac{n_{j0}}{n_{e0}} \left(\frac{m_j}{m_i}\right)^{\frac{1}{2}} \left(\frac{T_e}{T_j}\right)^{\frac{3}{2}} e^{-\frac{1}{2}\frac{T_e}{T_j}\frac{m_j}{m_i}\left(1-\frac{\omega^2}{\omega_{pi}^2}\right)}\right] \tag{23b}$$

for the complex k case. Since we consider the contamination effect, here, we have assumed $n_j << n_i \approx n_e$, where the ith ion species is the dominant species and the *j*th ion species is the impurity, and cs is the acoustic velocity of the dominant ions. Notice that the contamination does not affect the phase velocity of waves very much but it may affect the damping property significantly. From Eqs. (22*b*) and (23*b*), we find that the Landau damping is a summation of damping from electrons and all ion species. The ion Landau damping terms include the following function

$$f(x) = xe^{-\frac{1}{2}x^2}, \text{where } x = \sqrt{\frac{T_e}{T_j}\frac{m_j}{m_i}} \tag{24}$$

Figure 2 shows the function $f(x)$. Usually, the dominant ion species has $x > 1$. For the large T_e/T_i ratio, the value of $f(x)$ for the dominant ion species can be very small. The heavier ions (ions with masses greater than m_i) will have even smaller values. However, for the lighter ions (ions with masses less than m_i), the values of the function $f(x)$ and the Landau damping caused by them may have the same order as or even be greater than the Landau damping caused by

the dominant ions. Thus a small amount of light ion contamination may significantly affect the acoustic wave properties of a plasma. To study the contamination effect in detail, we consider the two-ion species plasma next. Assume that the contamination ion has mass $m_c < m_i$ and density n_c. Both ion species have the same temperature T_i. Following the method presented by *Fried et al.* [1979], we can get the approximate analytic expressions for the phase velocity and damping coefficient for the ion waves in the two-ion species plasma. For the high T_e/T_i ratio, we can obtain

$$\frac{\omega_r^2}{k^2} = c_s^2 F_{2\omega}(\vartheta), \tag{25a}$$

$$\left(\frac{\omega_i}{\omega_{pi}}\right)_L = -k\lambda_e \sqrt{\frac{\pi}{8}} \left(\frac{n_{e0}}{n_{i0}}\right)^{\frac{3}{2}} F_{2\omega}^2(\theta)\left[1+\alpha+\frac{6}{\theta}\left(1+\alpha\frac{m_i}{m_c}\right)F_{2\omega}^{-1}(\theta)\right]^{-1} G_{2\omega}(\theta) \tag{25b}$$

$$F_{2\omega}(\theta) = \frac{n_{i0}}{n_{e0}}\frac{1+\alpha}{1+k^2\lambda_e^2} + \frac{3}{\theta}\frac{1+\alpha(m_{i/}\, m_c)}{1+\alpha}$$

$$G_{2\omega}(\theta) = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \frac{n_{e0}}{n_{e0}}\left(\frac{m_c}{m_i}\right)^{\frac{1}{2}} \theta^{\frac{3}{2}} e^{\frac{\theta}{2}\frac{m_e}{m_i}F_{2\omega}(\theta)} + \frac{n_{i0}}{n_{e0}}\theta^{\frac{3}{2}} e^{-\frac{\theta}{2}F_{2\omega}(\theta)}$$

for the complex ω case, where

$$\alpha = n_{c0}m_i / n_{i0}m_c$$

and

$$\frac{\omega^2}{k_r^2} = c_s^2 F_{2k}(\theta) \tag{26a}$$

$$(k_i\lambda_e)_L = \frac{\omega}{\omega_{pi}}\sqrt{\frac{\pi}{8}}\left(\frac{n_{i0}}{n_{e0}}\right)^{\frac{1}{2}}\left[1+\frac{3}{\theta}\frac{n_{i0}}{n_{e0}}\left(1+\alpha\frac{m_i}{m_c}\right)F_{2k}^{-2}(\theta)\right]^{-1} G_{2k}(\theta) \tag{26b}$$

$$F_{2k}(\theta) = \frac{n_{i0}}{n_{e0}}\left(1+\alpha-\frac{\omega^2}{\omega_{pi}^2}\right) + \frac{3}{\theta}\frac{1+\alpha(m_i/m_c)}{1+\alpha}$$

$$G_{2k}(\theta) = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \frac{n_{c0}}{n_{e0}}\left(\frac{m_c}{m_i}\right)^{\frac{1}{2}} \theta^{\frac{3}{2}} e^{\frac{\theta}{2}\frac{m_c}{m_i}F_{2k}(\theta)} + \frac{n_{i0}}{n_{e0}}\theta^{\frac{3}{2}} e^{-\frac{\theta}{2}F_{2k}(\theta)}$$

for the complex k case.

For the medium T_e/T_i ratio, we can obtain

$$\frac{\omega_r^2}{k^2} = c_s^2 F_{3\omega}(\theta) \tag{27a}$$

$$\left(\frac{\omega_i}{\omega_{pi}}\right)_L = -k\lambda_e\sqrt{\frac{\pi}{8}}\left(\frac{n_{e0}}{n_{i0}}\right)^{\frac{3}{2}} F_{3\omega}^2(\theta)\left[1+\frac{6}{\theta}F_{3\omega}^{-1}(\theta)\right]^{-1} G_{3\omega}(\theta) \tag{27b}$$

$$F_{3\omega}(\theta) = \frac{n_{i0}}{n_{e0}}\frac{1}{1+\beta+k^2\lambda_e^2} + \frac{3}{\theta}$$

$$G_{3\omega}(\theta) = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \frac{n_{c0}}{n_{e0}}\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \theta^{\frac{3}{2}} e^{-\frac{\theta}{2}\frac{m_c}{m_i}F_{3\omega}(\theta)} + \frac{n_{i0}}{n_{e0}}\theta^{\frac{3}{2}} e^{-\frac{\theta}{2}F_{3\omega}(\theta)}$$

for the complex ω case, where

$$\beta = n_{c0}T_e / n_{e0}T_i;$$

and

$$\frac{\omega^2}{k^2} = c_s^2 F_{3k}(\theta) \tag{28a}$$

$$(k_i\lambda_e)_L = \frac{\omega}{\omega_{pi}}\sqrt{\frac{\pi}{8}}\left(\frac{n_{i0}}{n_{e0}}\right)^{\frac{1}{2}}\left[1+\beta+\frac{3}{\theta}\frac{n_{i0}}{n_{e0}}F_{3k}^{-2}(\theta)\right]^{-1} G_{3k}(\theta) \tag{28b}$$

$$F_{3k}(\theta) = \frac{n_{i0}}{n_{e0}}(1+\beta)^{-1}\left(1-\frac{\omega^2}{\omega_{pi}^2}\right)+\frac{3}{\theta}$$

$$G_{3k}(\theta) = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \frac{n_{c0}}{n_{e0}}\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}}\theta^{\frac{3}{2}}e^{-\frac{\theta m_e}{2 m_i}F_{3k}(\theta)} + \frac{n_{i0}}{n_{e0}}\theta^{\frac{3}{2}}e^{-\frac{\theta}{2}F_{3k}(\theta)}$$

for the complex k case. Note that *Fried et al.* [1971] only gave the analytic expression for the complex ω case which approximately are Eqs. (25) and (27). Figure 3 shows the dispersion relations of the ion acoustic waves in the argon plasma with the different ratios of H^+ contamination. From the figure we know that the phase velocity of waves becomes larger and larger as the contamination increases when the temperature ratio is small and becomes smaller and smaller when the temperature ratio is larger than certain value (about 0.011). The H^+ contamination has significant effect on the damping coefficient of the waves in the small temperature ratio region. It causes a peak in the damping curve of the waves. In the higher level of contamination, the situation becomes quite different. In Figure 3*b*, we show the case where the H^+ contamination is 1%. Now we find that at least two modes may exist for the waves: in the lower temperature ratio (the left side of the vertical dotted line), the mode 1 (solid line) has the lower damping coefficient and is the dominant mode; in the higher temperature ratio (the right side of the vertical dotted line), the mode 2 then has the lower damping coefficient and becomes the dominant mode. We also find that the dispersion relation of no H^+ contamination closes to the mode 2 in the high ion-electron temperature ratio and the mode 1 in the energy low temperature ratio. In the middle range of the temperature ratio , it gives neither approximate damping coefficient for the two modes. One interesting thing is that the wave phase velocity curves of the two modes have no intersecting point in this case. Thus, if we could control the temperature ratio of ion to electron, we might find that, at certain temperature, there will be two modes, with different phase velocity, propagate in the high H^+ contaminated plasma!

Actually, in the lower level of contamination, again there are at least two modes that exist, as we show in Figure 3*c* for the 0.5% H^+ contamination. The difference to higher contamination cases is that, now, the mode 1 (what we showed in Figure 3*a*) always has the lower damping coefficient than the mode 2 in the temperature range shown. From Figure 3d we know that the change from the lower-level contamination behavior to the higher-level one occurs around 0.65-0.7% of H^+ contamination.

The approximate formulas (25) and (27) are also as shown in Figure 3*b* and 3*c*. Eq. (25) does give a good approximation to the exact solution when the temperature ratio is very small but Eq. (26) does not when the temperature ratio is higher.

Figure 4 shows the similar situation to Figure 3 but for the complex k case. All the conclusions above almost keep the same except that the dominant mode and the non-dominant mode in this case will exchange in the very high temperature ratio (the right side of the second dotted line in Figure 4*b* and the right side of the dotted line in Figure 4*c*)

4. Experiments on Ion Acoustic Waves

The experiments of ion acoustic waves were performed in the double plasma chamber as shown in Figure 5. The typical plasma parameters used in the experiments are as follows: electron temperature T_e = 1~3eV, ion temperature $T_i \approx 0.05\sim0.1T_e$, electron density $n_e=10^8 \sim 10^{10}$ cm^{-3}, total neutral gas pressure p ≈ (1~2)x 10^{-4} Torr, and base pressure $p_b \leq 5$x 10^{-6} Torr. In the experiments, the argon gas was used. Figure 6 shows the gas composition in our chamber in the situations of the bass pressure and the operating pressure. From Figure 6a we find that the hydrogen, vapor, nitrogen, and oxygen ions will be the major light ion contaminations. Since our previous analysis has focused on the two- ion species plasma and our numerical solutions would also become unstable when too many ion species were considered, here we will count all the effects of light ion contaminations into one ion species. From Eq. (24) we can find the ratios of effects of the ion contaminations: H: N or O or H_2O: N_2: $O_2 \approx$ 0.5:0.08:0.014:0.007. So H^+ affects the most. Using these ratios and comparing he peak values of the various gases in Figure 6, we can get the equivalent H^+ contamination about 1.8%. In the experiments, waves were launched externally by the signal grid and were detected by the movable Langmuir probe. Figure 7 gives the waves we measured.

Since the experimental situations are considered here, the best suited case will be that k is complex and ω is real (complex k case). Figure 8 shows the results of wave number and damping coefficient obtained from the curve fitting. The solid lines in Figure 8 are the theoretical curves calculated from Eq. (13) with *e-n, e-e, i-i,* and *i-e* collision neglected. The parameters we used are: n_e=2x10^9 cm^{-3}, T_e = 1.4eV, T_i = 0.09eV, the neutral gas density n_n=4.62x10^{12} cm^{-3}, and the H^+ contamination 1.8%. From Figure 8 we find that the wave number is very close to the theoretical value. The damping coefficient is close to the theoretical value in the high frequency, but does not follow the theoretical predication when the frequency is less than 60kHz. Two reasons may contribute to this disagreement: first, the wavelength is near the size of our chamber; and second, this disagreement starts at the wave frequency which is only several times higher than the e-i collision frequency which is about 10kHz. We know that at low frequency the general propagator expansion method cannot give good approximation.

Table 1 shows the distributions of the Landau damping and the collision damping (or undamping) effects in the lower frequency ($\omega \ll \omega_{pi}$). In the table we also show the approximate values, in which the Landau damping is from Eq. (28*b*), the *i-n* collision damping and the *e-i* collision growth is from Eq. (20*b*). From Table 1, we find that them approximate formulas for Landau damping and the *i-n* collision damping is not precise

enough and from the discussion in the preceding section we know that the approximate formulas for the Landau damping is not good when the ion-to-electron temperature ratio is large. The approximate formula for the *i-n* collision damping is mainly affected by the contaminating effect. Note that, at lower frequency, the Landau damping coefficient is approximately proportional to the frequency and the damping coefficients of the *e-i* and *i-n* collision are approximately independent of the frequency. Since the *e-i* collisional growth is much higher than the *i-n* collisional damping for the plasma parameters we used, the theory predicates that the instability caused by the *e-i* collision might happen at low frequency as the solid line in Figure 8 showed. We did not find this instability in our experiment because of the reasons we mentioned before. This instability may be found in the experiment with low Landau damping.

Table 1. Distributions of damping or undamping effect in lower frequency

	Actual values	Approximate values
Landau damping only	$0.064\ (\omega/\omega_{pi})$	$0.076(\omega/\omega_{pi})$
e-i collision only	-10^{-3}	-10^{-3}
i-n collision only	10^{-4}	$4\text{x}10^{-5}$
total	$-9\text{x}10^{-4}+0.064(\omega/\omega_{pi})$	$-10^{-3}+0.076(\omega/\omega_{pi})$

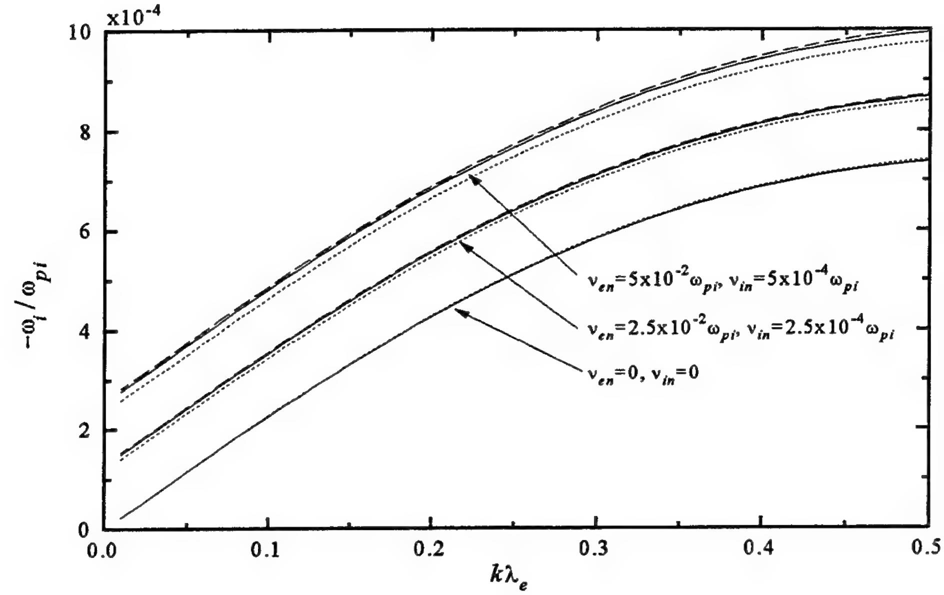
5. Conclusion

In this report, we have studied the effects of collisions and contamination on the ion acoustic waves.

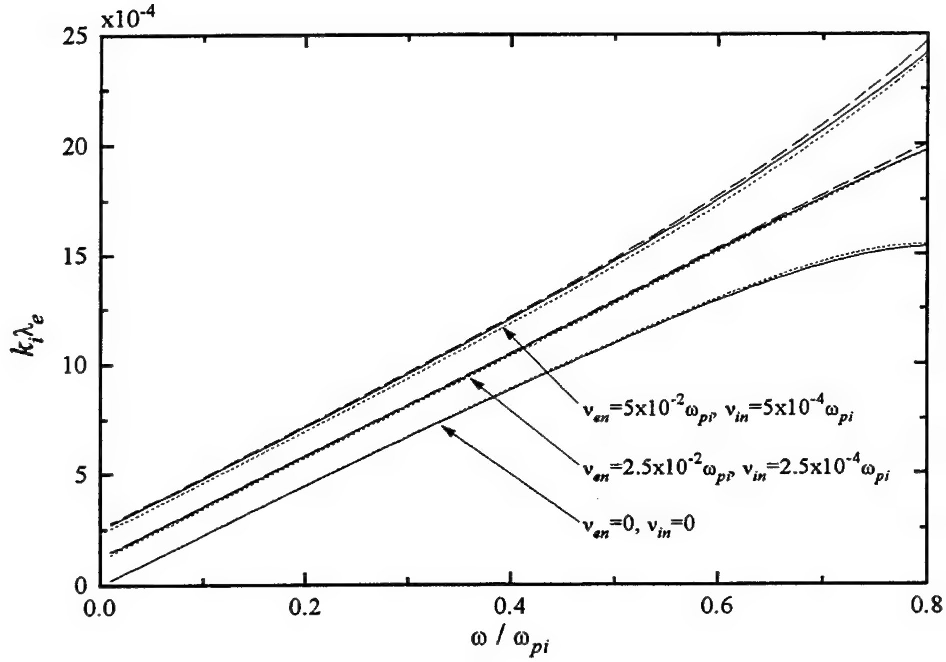
When collisions are added to the ion acoustic waves, the characteristic of the ions acoustic waves may change. In this report, we consider the *e-n, i-n, e-e, e-i, i-i,* and *i-e* collisional effects on the ion acoustic waves. We find that the *e-n* collision is weak compared to *i-n* collision and the *e-e, i-i,* and *i-e* collisions are weak compared to the *e-i* collision. The *i-n* collision are damped but the *e-i* collision may grow. The growth of the waves may happen when the *e-i* collision growth overcomes the Landau damping and other collision dampings and then the waves are unstable.

The contamination may have a significant effect on the ion acoustic waves. A small amount of light ion contamination can cause much higher damping and the wave property may be a very complicated function of ion composition and temperature.

In our experiment on the ion acoustic waves in a drift-free plasma, the argon gas was used. The experimental results are explained with the theory discussed in this report and are very close to our theoretical results.

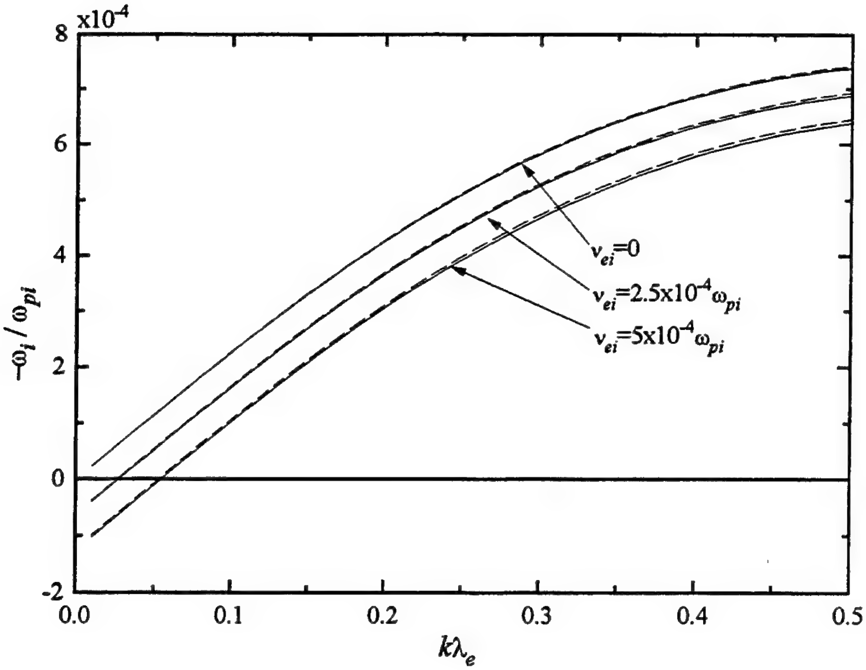


(a)

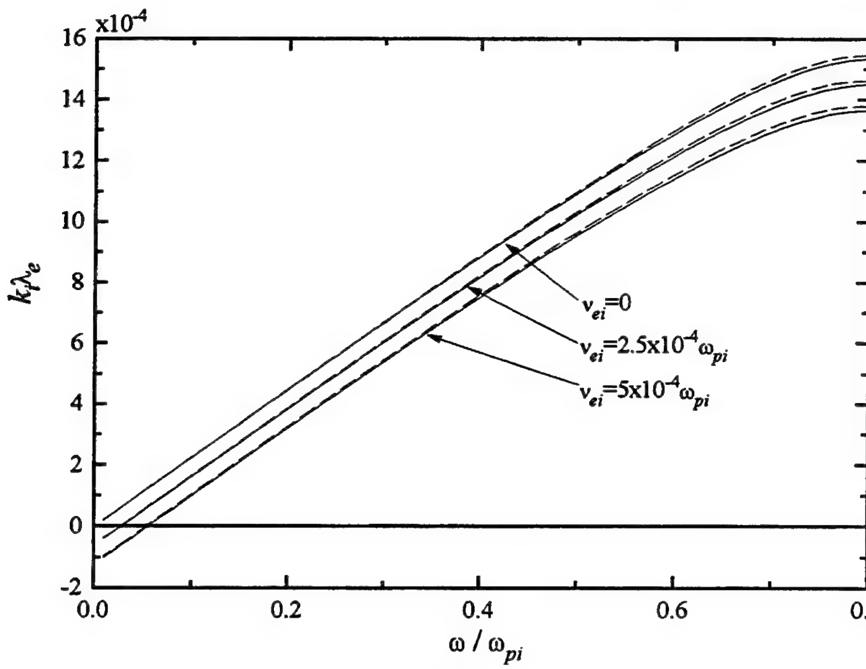


(b)

Figure 1*a*. Normalized damping coefficient versus (a) normalized wave number (complex ω case), and (b) normalized wave angular frequency (complex k case) for argon plasma. The values T_i/T_e=0.01 is used.



(a)



(b)

Figure 1*b*. Normalized damping coefficient versus (a) normalized wave number (complex ω case), and (b) normalized wave angular frequency (complex k case) for argon plasma. The values T_i/T_e=0.01 is used.

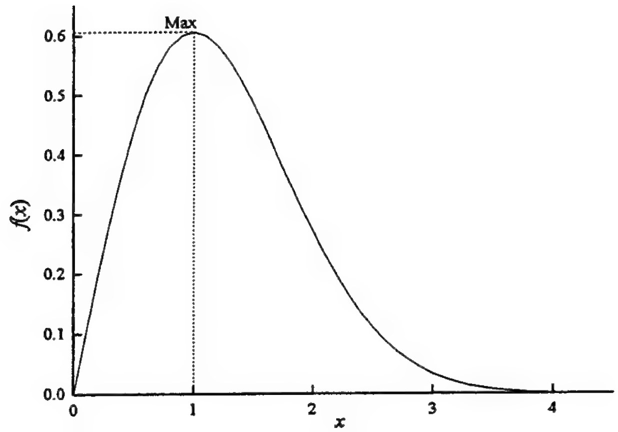


Figure 2. A function in the damping coefficient of ion acoustic waves in the multi-ion species plasma.

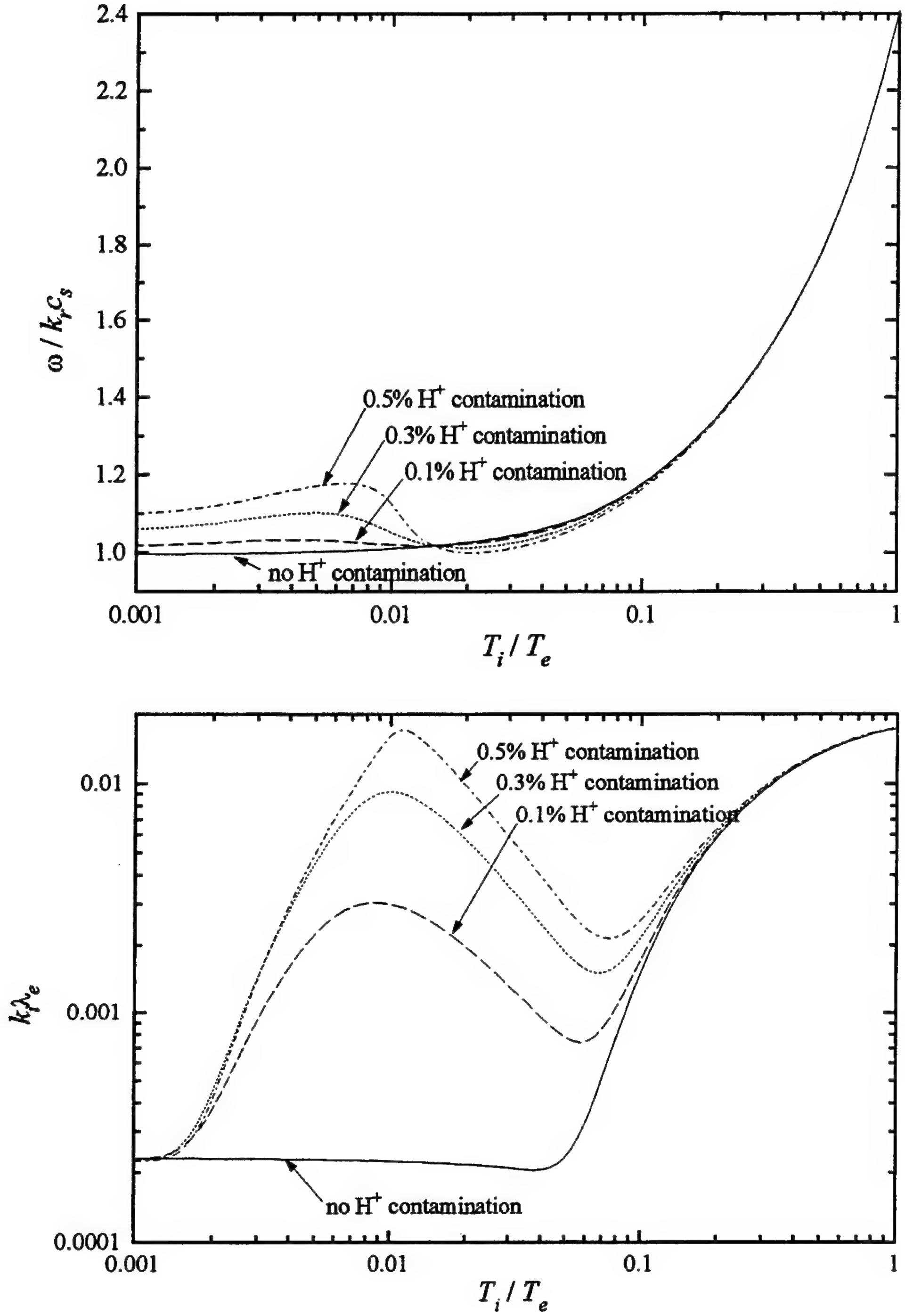


Figure 3a: Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H^+ contamination (complex ω case). Lines are computed from Eq. (13). The value $k\lambda_e = 0.1$ is used.

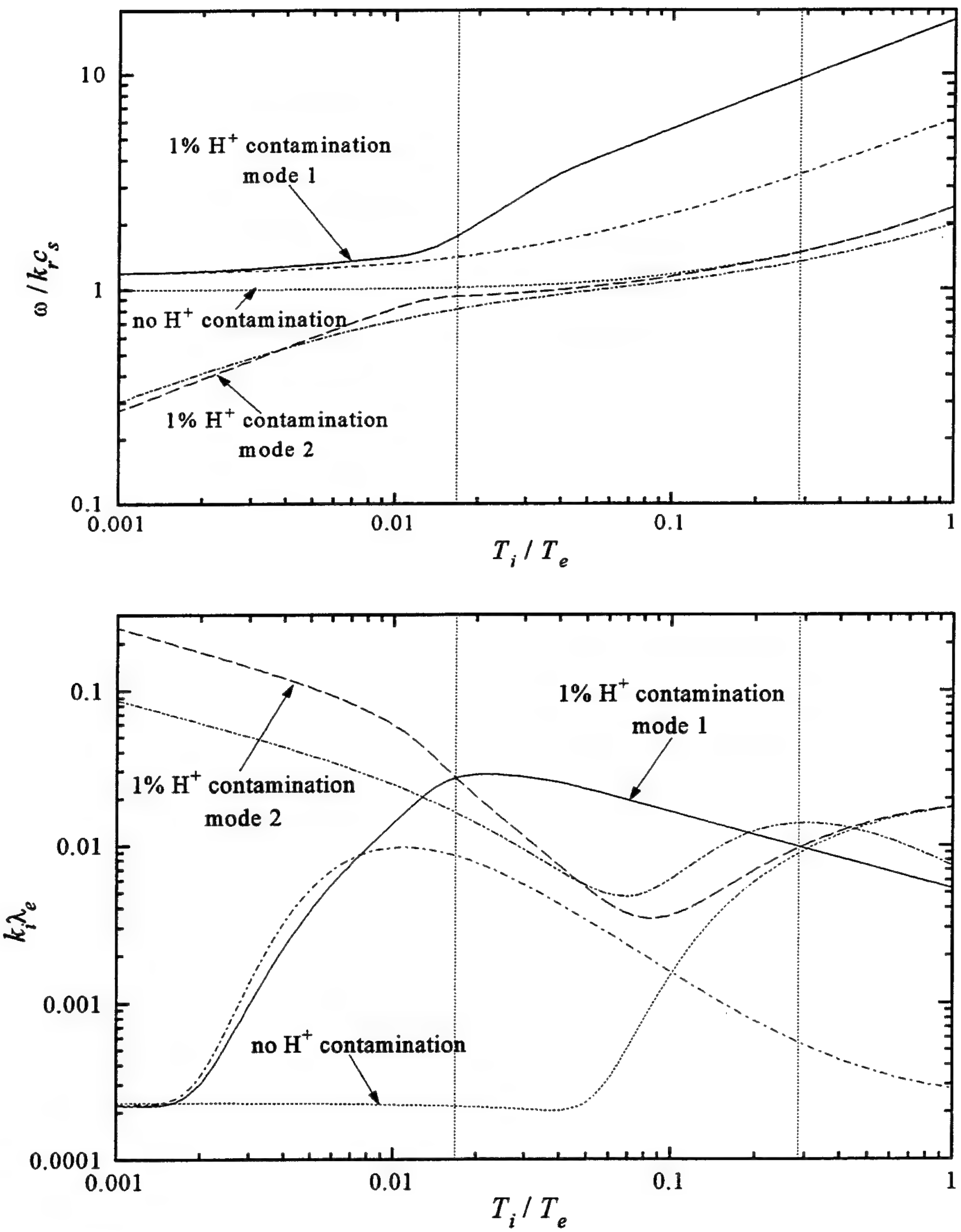


Figure 3*b*. Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H^+ contamination (complex ω case). The dot-and-dash and the dot-dot-and-dash lines are computed from Eqs. (25) and (27), respectively.

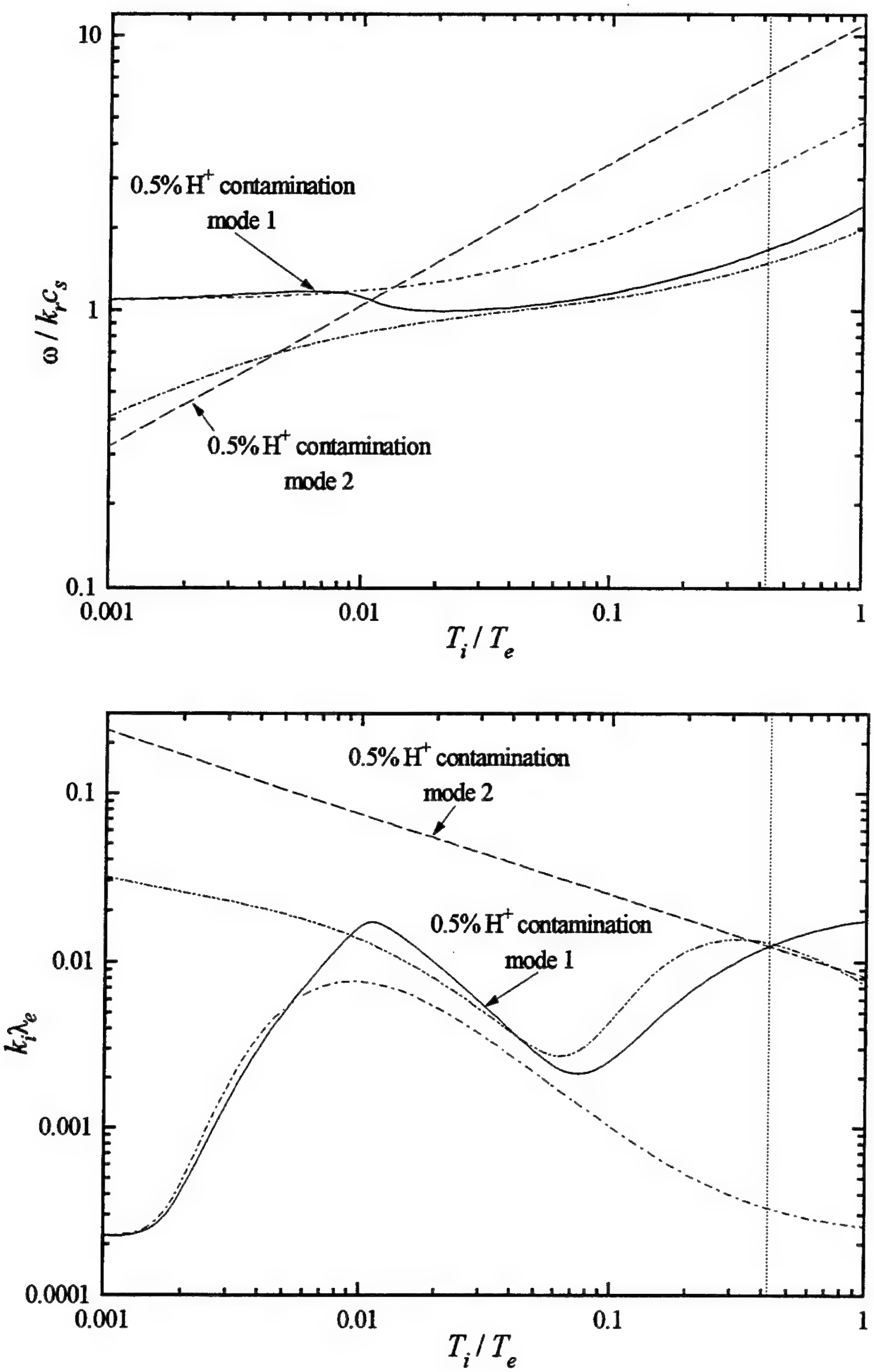
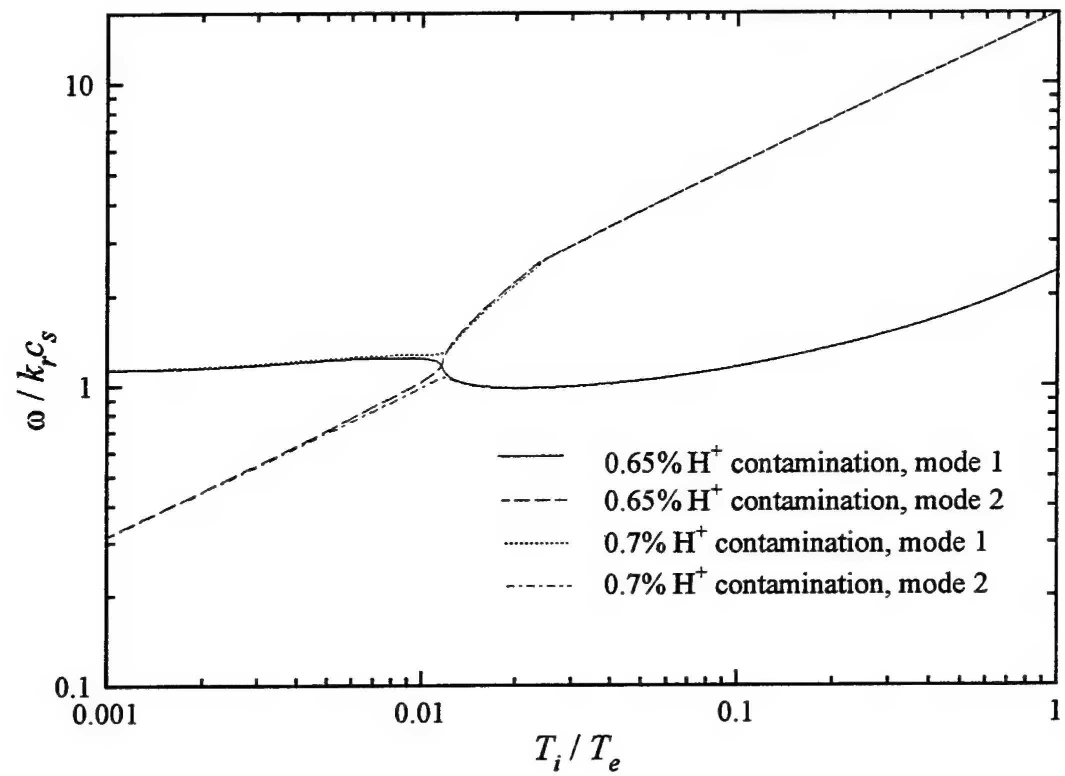


Figure 3*c*. Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H^+ contamination (complex ω case). The dot-and-dash and the dot-dot-and-dash lines are computed from Eqs. (25) and (27), respectively.



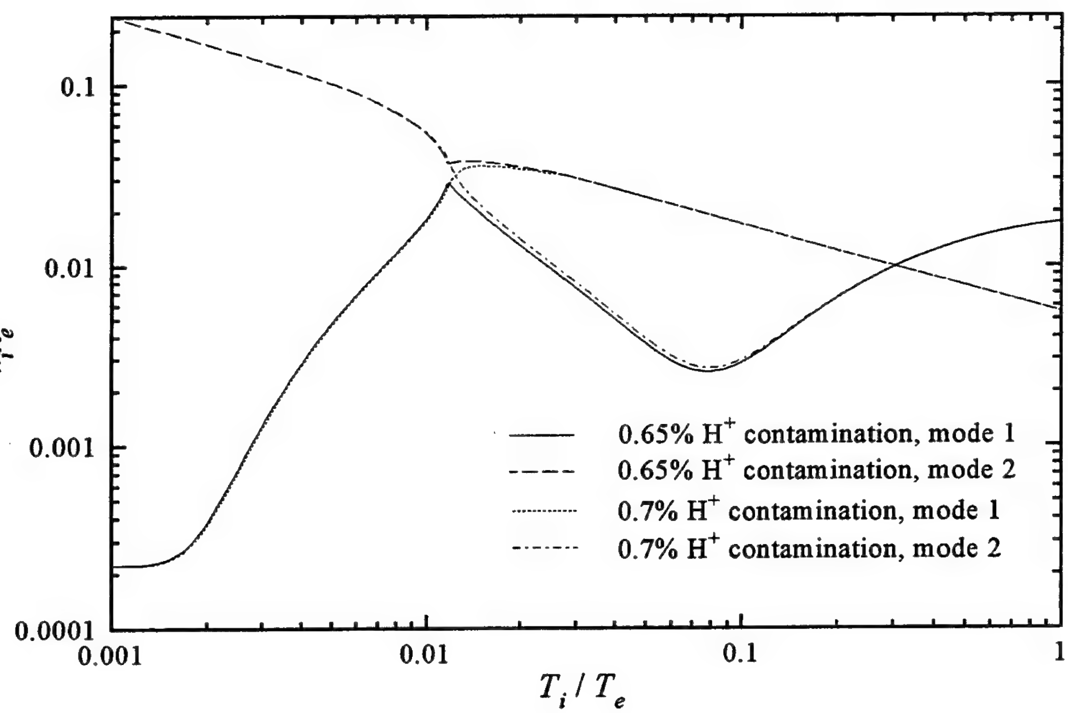


Figure 3*d*. Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H^+ contamination (complex ω case). Lines are computed from Eq. (13). The value $k\lambda_e = 0.1$ is used.

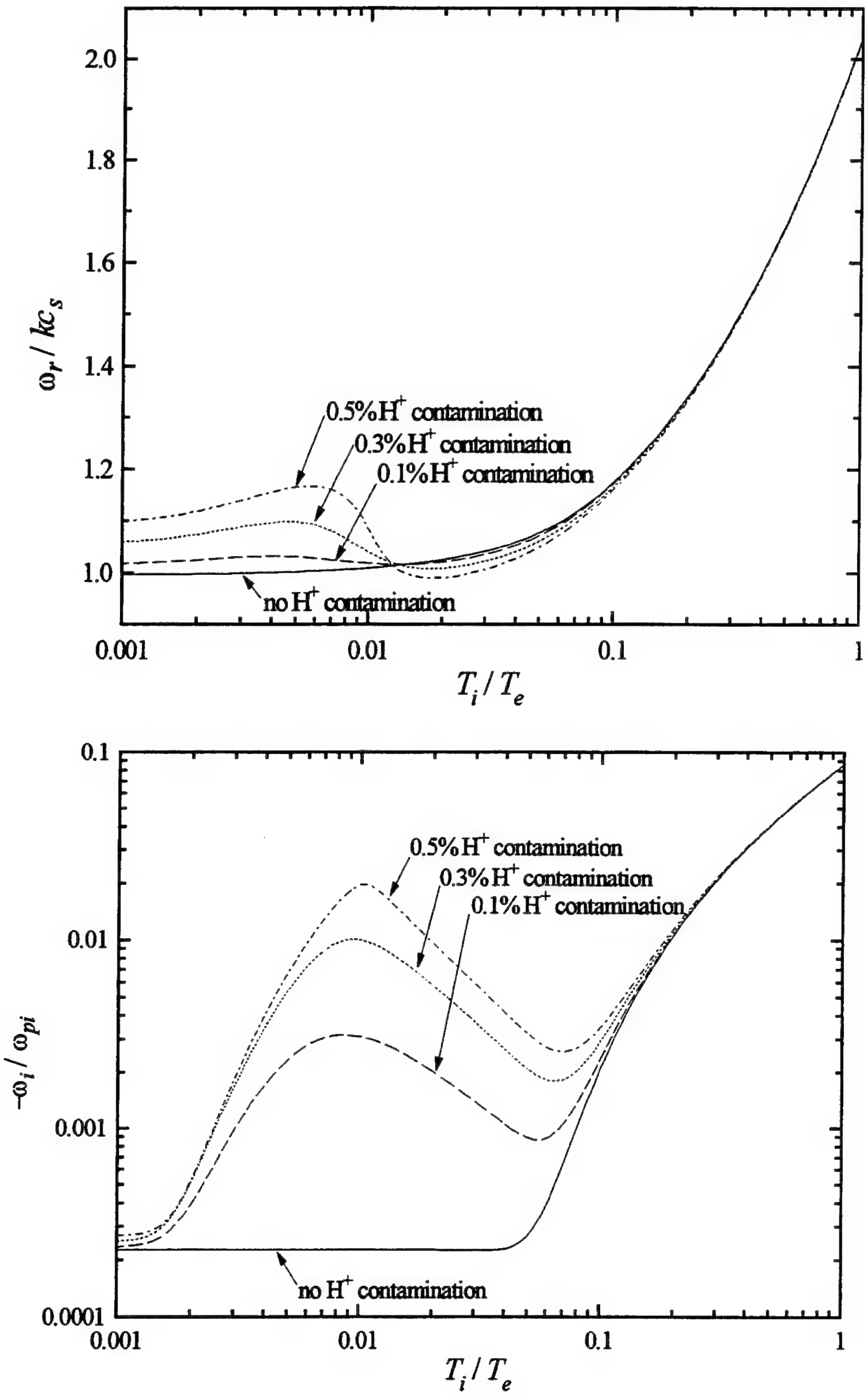


Figure 4*a*. Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H$^+$ contamination (complex$k$ case). Lines are computed from Eq. (13). The value $k\lambda_e = 0.1$ is used.

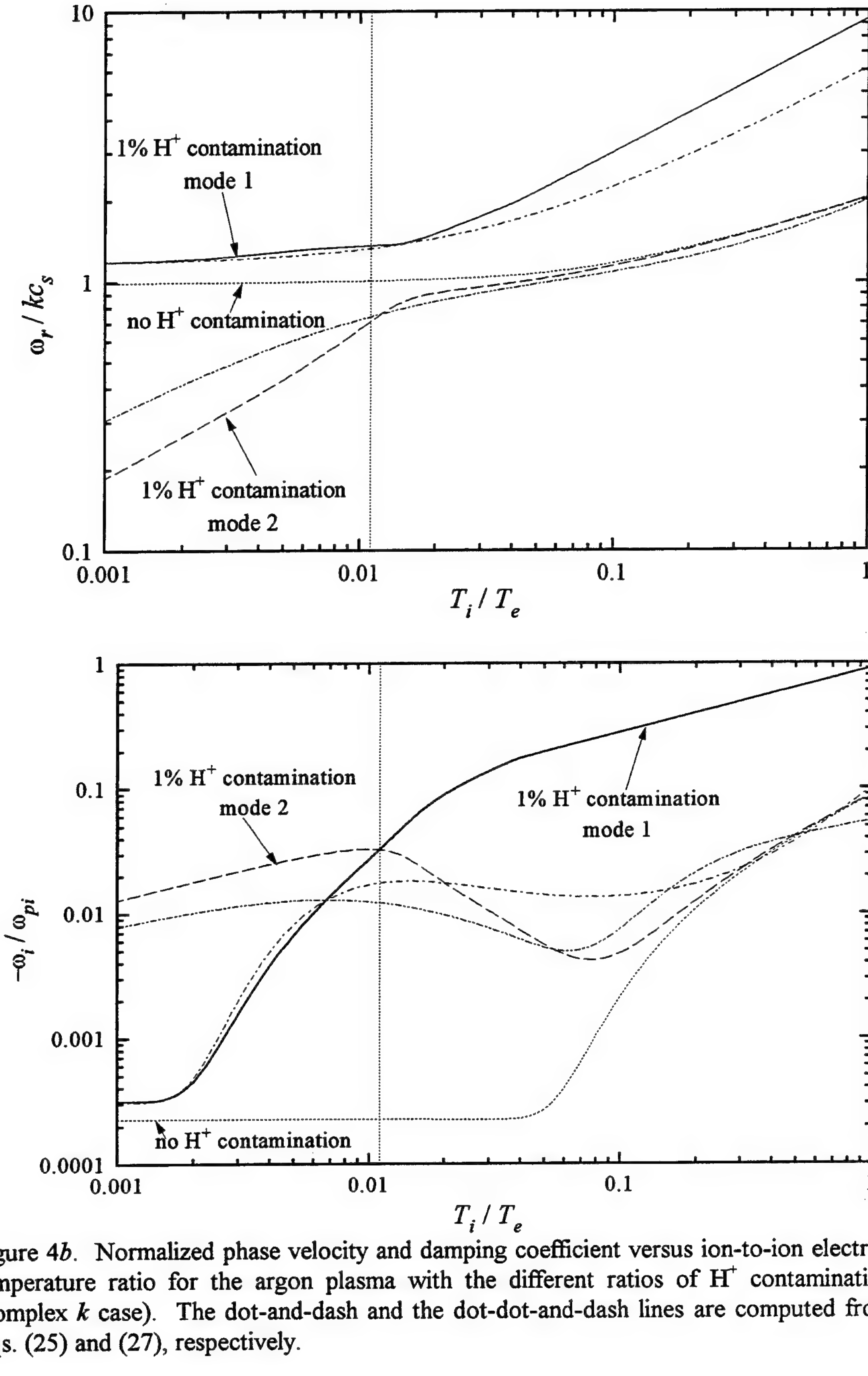


Figure 4*b*. Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H$^+$ contamination (complex k case). The dot-and-dash and the dot-dot-and-dash lines are computed from Eqs. (25) and (27), respectively.

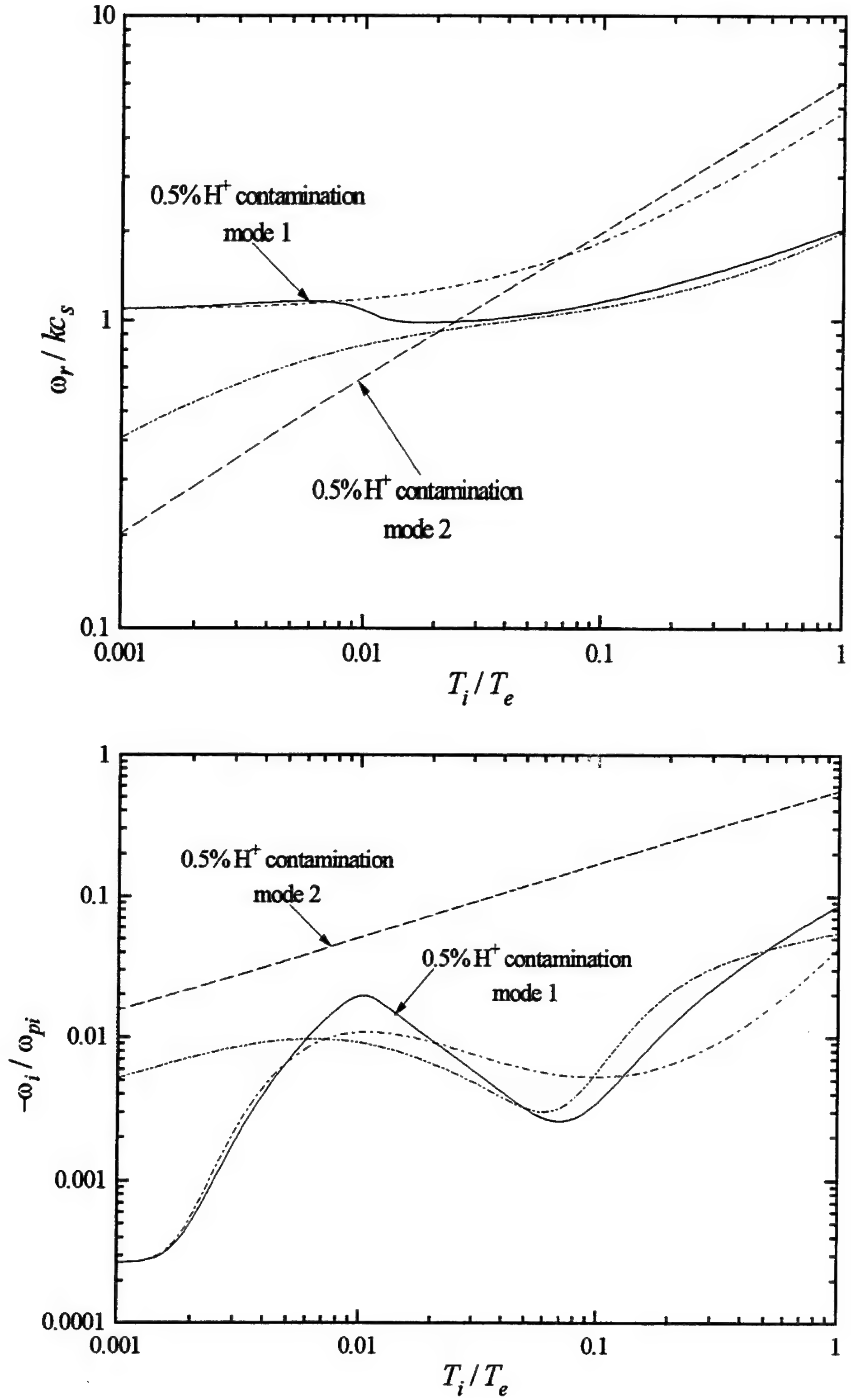


Figure 4*c*. Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H$^+$ contamination (complex *k* case). The dot-and-dash and the dot-dot-and-dash lines are computed from Eqs. (25) and (27), respectively.

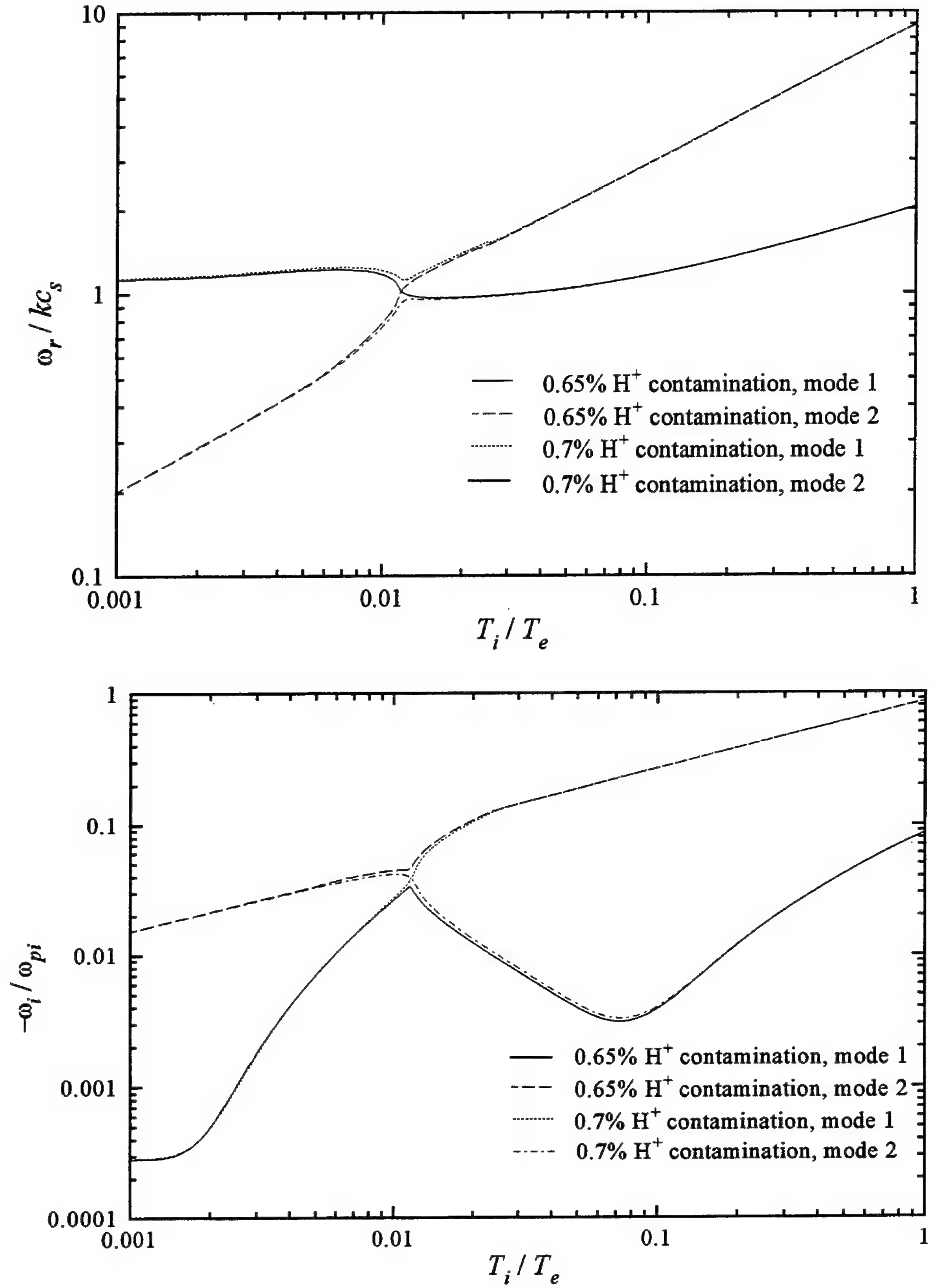
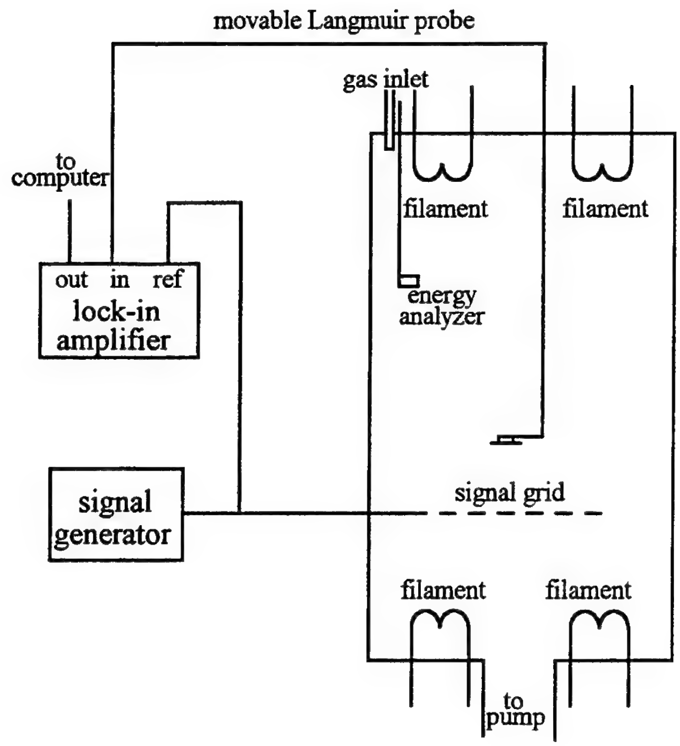
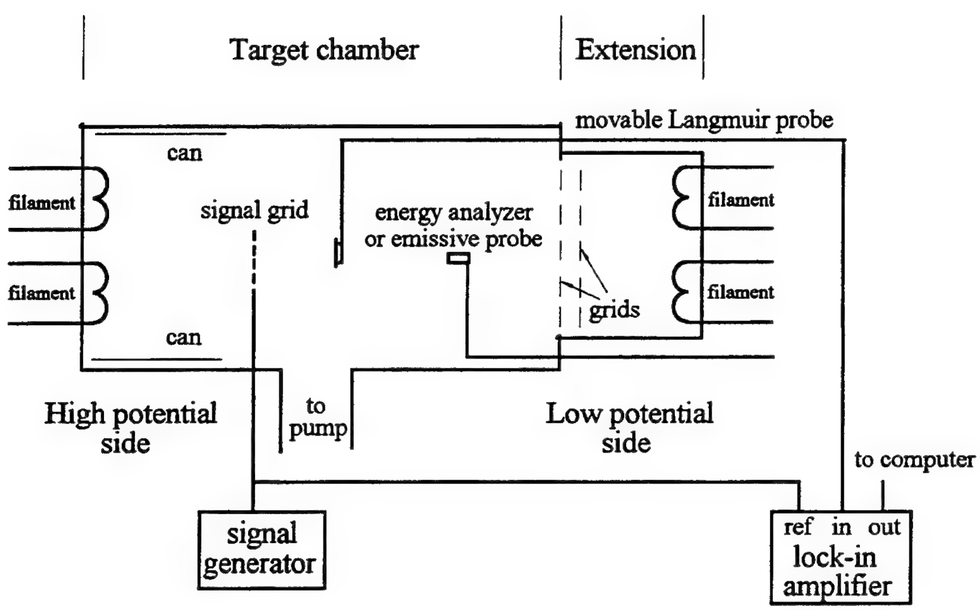


Figure 4*d*. Normalized phase velocity and damping coefficient versus ion-to-ion electron temperature ratio for the argon plasma with the different ratios of H$^+$ contamination (complex k case). Lines are computed from Eq. (13). The value $k\lambda_e = 0.1$ is used.

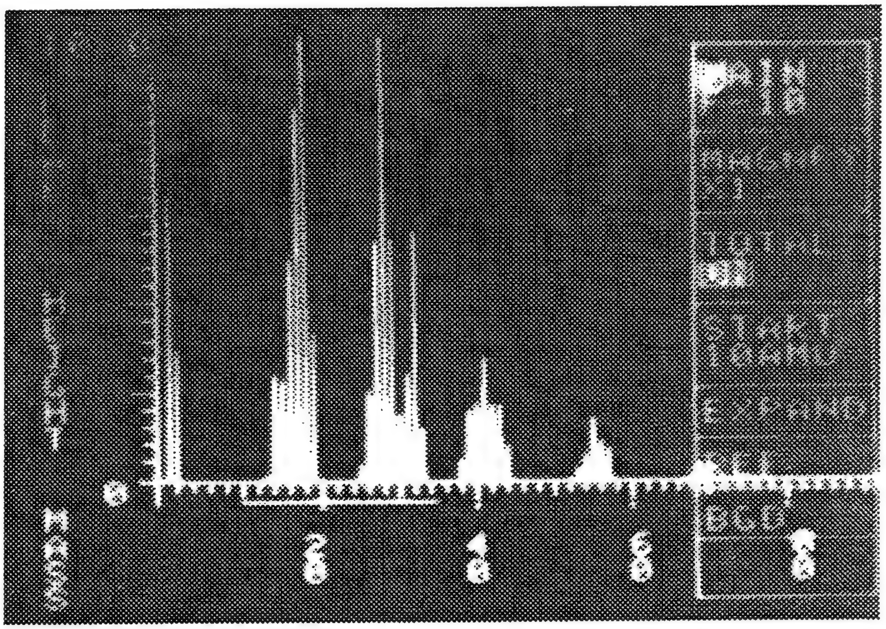


(a)

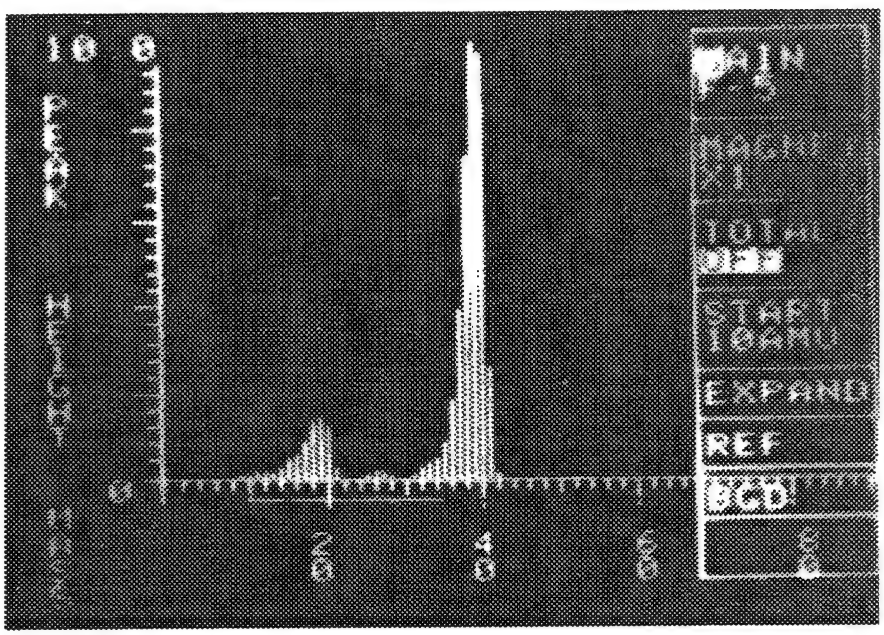


(b)

Figure 5. Schematic of two chambers and measuring systems. (a). Chamber I, the double plasma chamber; (b). Chamber II, the modified triple-layer plasma chamber.



(a)



(b)

Figure 6. The gas composition taken by MICROMASS. (a). In the situation of base pressure p_b=4×10^{-6} Torr; (b). In the situation of argon gas pressure p=1.32×10^{-4} Torr. Note that the argon gas also has a minor peak at 20AMU with height of 16% of its major peak.

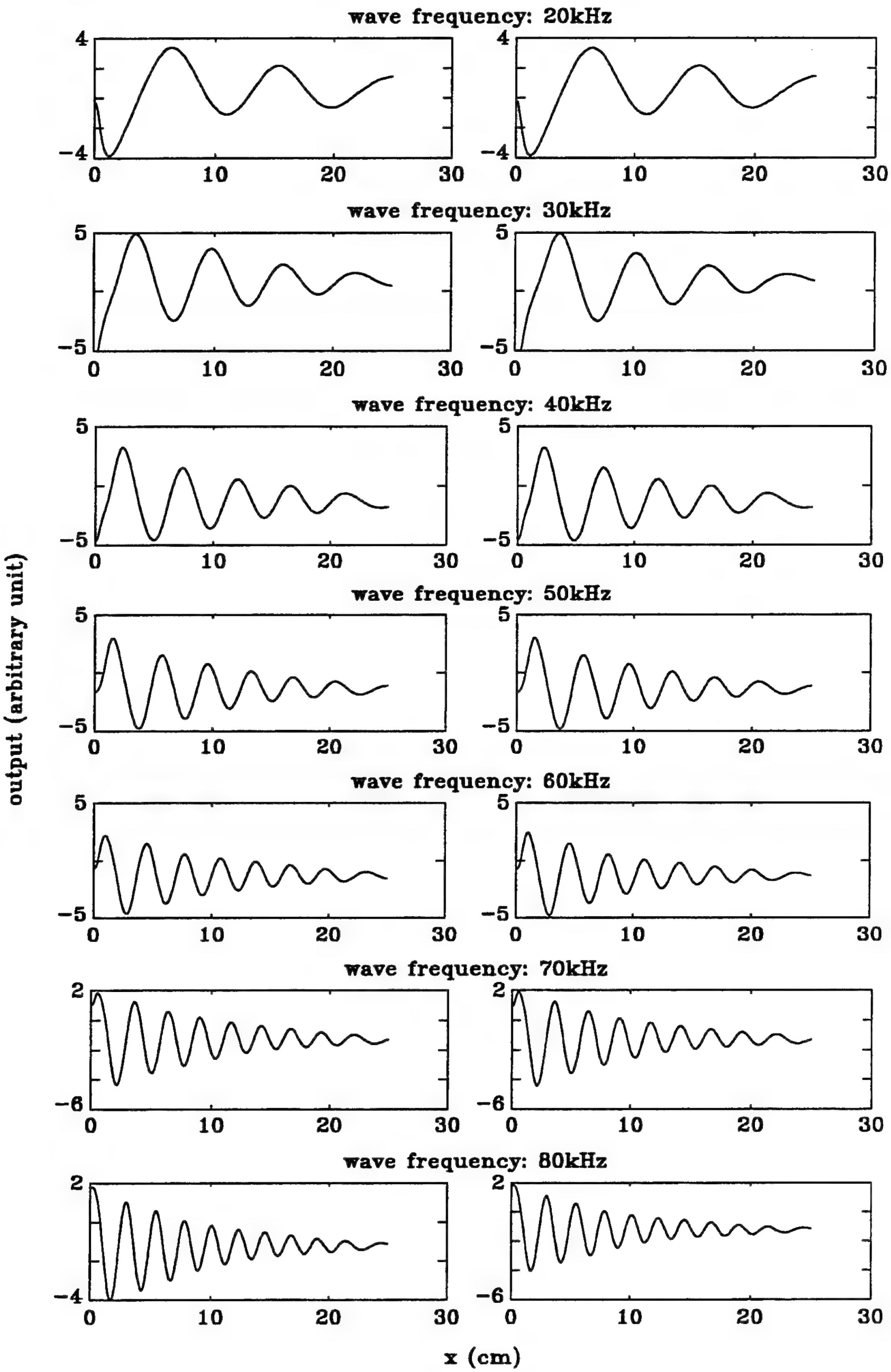


Figure 7. The ion acoustic waves measured in different frequencies.

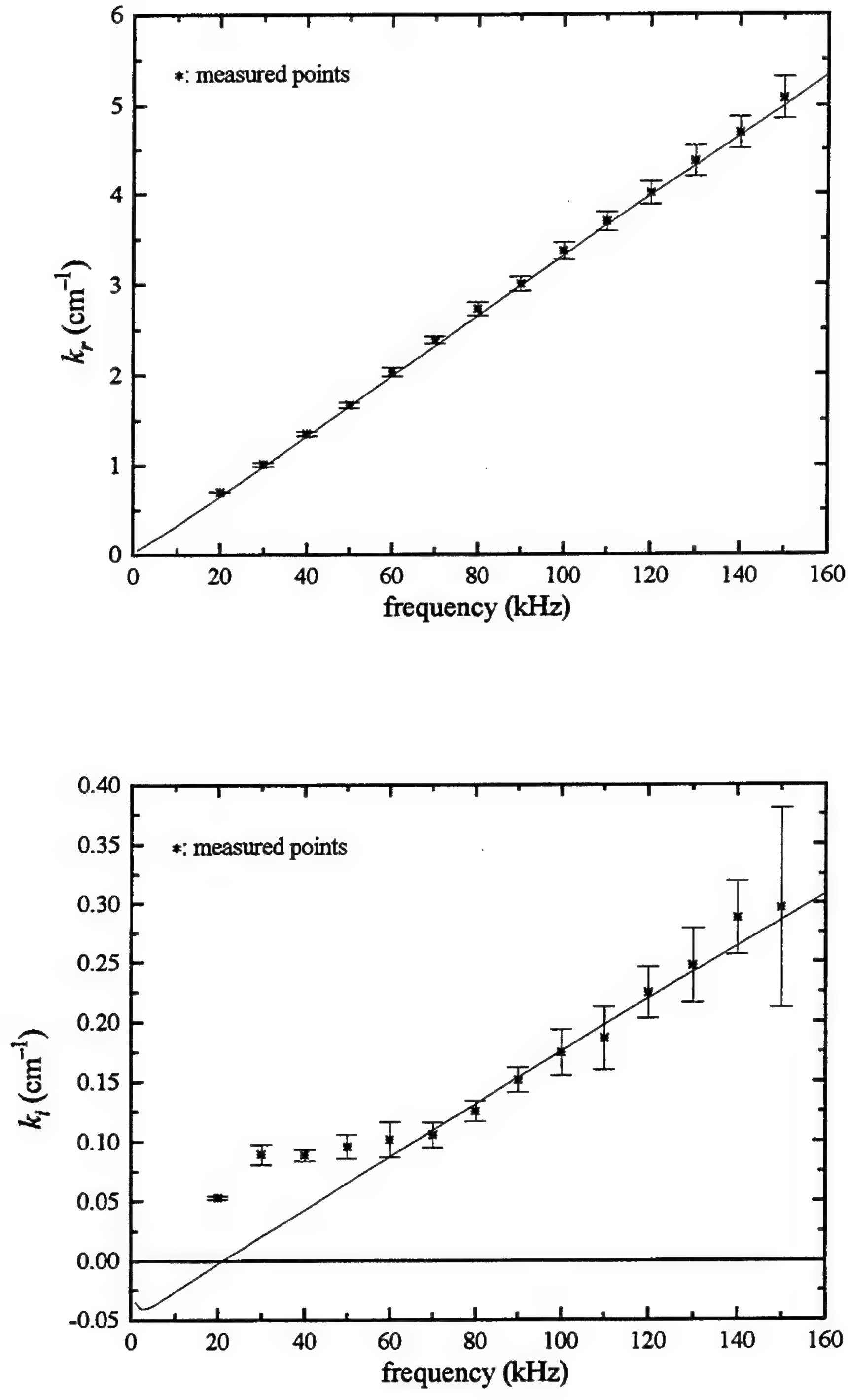


Figure 8. The wave number and the damping coefficient of ion acoustic waves versus frequency. The solid lines are the theoretical curves.

Appendix A. Special Functions for Dispersion Relations of Plasma Waves

A.1. Plasma Dispersion Function

The plasma dispersion function is defined as

$$Z'(\zeta) \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\xi^2}}{\xi - \zeta} d\xi \tag{A.1}$$

for $\Im(\zeta) > 0$ and as the analytic continuation of Eq. (A.1) for $\Im(\zeta) \leq 0$ [*Fried and Conte*, 1961].

The plasma dispersion function satisfies the differential equation

$$Z'(\zeta) = -2[1 + \zeta Z(\zeta)] \tag{A.2}$$

with $Z(0) = i\sqrt{\pi}$.

$Z(\zeta)$ can be expressed in terms of the error function

$$Z(\zeta) = i\sqrt{\pi} e^{-\zeta^2} \left[1 - \mathrm{erf}(i\zeta)\right] \tag{A.3}$$

where

$$\mathrm{erf}(\zeta) = \frac{2}{\sqrt{\pi}} \int_0^{\zeta} e^{-t^2} dt$$

is the error function. It should be noted that Eq. (A.3) is analytically continuous in the whole complex plane.

For the purpose of mathematical analysis, it is useful to find the series expansions of the plasma dispersion function for small and large arguments. For small arguments, $Z(\zeta)$ can be expressed in convergent series

$$Z(\zeta) = i\sqrt{\pi} e^{-\zeta^2} - \sum_{n=0}^{\infty} (-1)^n \frac{2^{n+1} \zeta^{2n+1}}{(2n+1)!!}, \tag{A.4}$$

and

$$Z(\zeta) = i\sqrt{\pi} e^{-\zeta^2} - 2e^{-\zeta^2} \sum_{n=0}^{\infty} \frac{\zeta^{2n+1}}{n!(2n+1)}. \tag{A.5}$$

For large arguments, we have the asymptotic series

$$Z(\zeta) = i\sqrt{\pi}\sigma e^{-\zeta^2} - \sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n \zeta^{2n+1}}, \tag{A.6}$$

$$\sigma = \begin{cases} 0, & \Im(\zeta) > 0 \\ 1, & \Im(\zeta) = 0 \quad and (-1)!! = 1 \\ 2, & \Im(\zeta) < 0 \end{cases}$$

The asymptotic series must be terminated when the terms no longer decrease.

A.2. W Function

An equivalent function to the plasma dispersion function is the W function, which is defined by *Ichimaru* [1973]

$$W_0(\zeta) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\xi e^{-\frac{\zeta^2}{2}}}{\xi - \zeta} d\xi \tag{A.7}$$

for $\Im(\zeta) > 0$ and as the analytic continuation of Eq. (A.7) for $\Im(\zeta) \leq 0$.

The W function satisfies the differential equation

$$W_0'(\zeta) = \zeta^{-1}[W_0(\zeta) - 1] - \zeta W_0(\zeta) \tag{A.8}$$

with $W_0(0) = 1$.

The W function is related to the plasma dispersion function either by

$$W_0(\zeta) = -\frac{1}{2} Z'\left(\frac{\zeta}{\sqrt{2}}\right) \tag{A.9}$$

or by

$$Z'(\zeta) = -2W_0\left(\sqrt{2}\zeta\right). \tag{A.10}$$

The higher order derivative of the W function is defined as

$$W_n(\zeta) = \frac{d^n}{d\zeta^n} W_0(\zeta) = i^n \int_0^\infty t^{n+1} e^{i\zeta t - \frac{t^2}{2}} dt. \tag{A.11}$$

Notice that the above definition is also valid when n=0. The higher order derivative of the W function can also be calculated from the lower order functions

$$W_0(\zeta) = 1 - \zeta W_{-1}(\zeta), \tag{A.12a}$$

$$W_n(\zeta) = -nW_{n-2}(\zeta) - \zeta W_{n-1}(\zeta). \tag{A.12b}$$

In Eq. (A.12a), we introduce W-1(ζ). The function W-1(ζ) can be defined from Eq. (A.11) as

$$W_{-1}(\zeta) = -i \int_0^\infty e^{i\zeta t - \frac{t^2}{2}} dt. \tag{A.13}$$

Obviously, W0(ζ) and W-1(ζ) satisfies

$$W_0(\zeta) = \frac{d}{d\zeta} W_{-1}(\zeta) \tag{A.14}$$

and W(ζ) is related to Z(ζ) by

$$W_{-1}(\zeta) = -\frac{1}{\sqrt{2}} Z\left(\frac{\zeta}{\sqrt{2}}\right). \tag{A.15}$$

Following are three series expansions for the W_{-1} (ζ) which correspond to Eqs. (A.4), (A.5), and (A.6) for the plasma dispersion function. They are

$$W_{-1}(\zeta) = -i\sqrt{\frac{\pi}{2}} e^{-\frac{\zeta^2}{2}} + \sum_{n=0}^{\infty} (-1)^n \frac{\zeta^{2n+1}}{(2n+1)!!}, \tag{A.16}$$

and

$$W_{-1}(\zeta) = -i\sqrt{\frac{\pi}{2}} e^{-\frac{\zeta^2}{2}} + e^{-\frac{\zeta^2}{2}} \sum_{n=0}^{\infty} \frac{\zeta^{2n+1}}{(2n)!!(2n+1)} \tag{A.17}$$

for small arguments, and

$$W_{-1}(\zeta) = -i\sqrt{\frac{\pi}{2}} \sigma e^{-\frac{\zeta^2}{2}} + \sum_{n=0}^{\infty} \frac{(2n-1)!!}{\zeta^{2n+1}}, \tag{A.18}$$

for large arguments.

A.2 Modified Bessel Function of the First Kind

The solution to the differential equation

$$\zeta^2 \frac{d^2 w}{d\zeta^2} + \zeta \frac{dw}{d\zeta} - \left(\zeta^2 + \mathrm{v}^2\right) w = 0 \tag{A.19}$$

are $I_{\pm \mathrm{v}}(\zeta)$ and $K_{\mathrm{v}}(\zeta)$. The function $I_{\mathrm{v}}(\zeta)$ is referred to as the modified Bessel function of the first order v [*Abramowitz and Stegun*, 1972].

Some of the identities involving the modified Bessel function of the first kind are

$$I_{-n}(\zeta) = I_N(\zeta), n = 0,\ 1, 2, 3, \ldots \tag{A.20}$$

$$e^{\zeta \cos\phi} = \sum_{n=-\infty}^{\infty} I_n(\zeta) e^{in\phi}, \tag{A.21}$$

$$\frac{2\mathrm{v}}{\zeta} I_{\mathrm{v}}(\zeta) = I_{\mathrm{v}-1}(\zeta) - I_{\mathrm{v}+1}(\zeta), \tag{A.22}$$

$$2I_{\mathrm{v}}'(\zeta) = I_{\mathrm{v}-1}(\zeta) + I_{\mathrm{v}+1}(\zeta). \tag{A.23}$$

For small arguments the modified Bessel function of the first kind can be computed using the series of

$$I_{\mathrm{v}}(\zeta) = \sum_{n=0}^{\infty} \frac{(\zeta/2)^{2n+\mathrm{v}}}{\mathrm{n}!\Gamma(n+\mathrm{v}+1)}; \tag{A.24}$$

and for large arguments, using the asymptotic expansion of

$$I_{\mathrm{v}}(\zeta) = \frac{e^{\zeta}}{\sqrt{2\pi\zeta}}\left[1 - \frac{\mu-1}{8\zeta} + \frac{(\mu-1)(\mu-9)}{2!(8\zeta)^2} - \frac{(\mu-1)(\mu-9)(\mu-25)}{3!(8\zeta)^3} + \ldots\right], |\arg\zeta| < \frac{\pi}{2}, \tag{A.25}$$

where

$$\mu = 4\mathrm{v}^2.$$

References

Alexeff, L, W.D. Jones, and D. Montgomery, Controlled Landau damping of ion-acoustic waves, *Phys. Rev. Lett.*, 19, 422-425, 1967.

Basu, B, and J.R. Jasperse, Marginal stability of ion-acoustic waves in a weakly collisional two-temperature plasma without a current, *Phys. Rev.*, 38, 3785-3788, 1988.

Bhadra, D., and R. K. Varma, Collisional damping of ion waves in a plasma, *Phys. Fluids*, 7, 1091-1093, 1964.

Buti, B., Ion acoustic waves in a collisional plasma, *Phys. Rev.*, 165, 195-201, 1968.

Chandrasekhar, S., Stochastic problems in physics and astronomy, *Rev. Mod. Phys.*, 15, 1-89, 1943.

Fokker, A. D., *Ann. d. Physik*, 43, 812, 1914.

Fried, B. D. and S. D. Conte, *The Plasma Dispersion Function*, Academic Press, New York and London, 1961.

Fried, B. D., R. B. White, and T. K. Samec, Ion acoustic waves in a multi-ion plasma, *Phys. Fluids*, 14, 2388-2392, 1971.

Grewal, M. S., Comparison of Fokker-Planck and Bhatnagar-Gross-Krook equations, *Phys. Rev.*, 136, A1181-A1186, 1964.

Jasperse, J. R., A propagator expansion method for solving linearized plasma kinetic equations with collisions, *Phys. Lett.*, 106A, 379-382, 1984.

Jin, Z., J. Hamila, R. C. Allen, S. Meassick, and C. Chan, Laboratory experiments on particle acceleration processes associated with parallel electric fields, in *Space Plasma: Coupling Between Small and Medium Scale Processes, Geophys. Monogr. Ser.*, 86, edited by M. Ashour-Abdalla et al., AGU, Washington, D.C., 95-98, 1995.

Kulsrud, R. M., and C. S. Shen, Effect of week collisions on ion waves, *Phy. Fluids*, 9, 177-186, 1966.

Landau, L. D., On the vibration of the electron plasma, *J. Phys.* (USSR), 10, 25, 1946.

Liboff, R. L., *Kinetic Theory: Classical Quantum, and Relativistic Descriptions*, Prentice Hall, Englewood Cliffs, NJ, 1990.

Nakamura, M., M. Ito, Y. Nakamura, and T. Itoh, Maximization in the damping rate of the ion acoustic wave in a two-ion plasma, *Phys. Fluids*, 18, 651-655, 1975.

Nakamura, Y., M. Nakamura, and T. Itoh, Observation of two ion-acoustic waves in an argon-helium plasma, *Phys. Rev. Lett.*, 37, 209-212, 1976.

Ono, M., and R. M. Kulsrud, Frequency and damping of ion acoustic waves, *Phys. Fluids*, 18, 1287-1293, 1975.

Planck, M., *Sitz. der Preuss. Akad.*, 324, 1917.

Revans, R. W., The transmission of waves through an ionized gas, *Phys. Rev.*, 44, 798-802, 1933.

Rosenbluth, M. N., M. MacDonald, and D. L. Judd, Fokker-Planck equation for an inverse-square force, *Phys. Rev.*, 107, 1-6, 1957.

Stéfant, R. J., Influence of electron-ion collisions on ion acoustic waves, *Phys. Fluids*, 14, 2245-2246, 1971.

Wong, A. Y., R. W. Motley, and N. D'Angelo, Landau damping of ion acoustic waves in highly ionized plasmas, *Phys. Rev.*, 133, A436-A442, 1964.